International Journal of Management, IT & Engineering
Vol. 9 Issue 5, May 2019,
ISSN: 2249-0558 Impact Factor: 7.119
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gate as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Semi-T_0 Spaces of Semi-Separation axioms in topological space

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Abstract : In this paper, we introduce a new class of space in the topological space, namely Semi-T_0 space of Semi-Separation axioms in the topological space. We find characterizations of these spaces. Further, we study some fundamental properties of these spaces in the topological space.

Keywords : Semi-open set, Semi-closed set, Semi-closure.

I. Introduction

The term Semi-T_0 space is weaker form of T_0 space. It plays a significant role in the topological space. Ever since the concept of the term Semi-T_0 space & Semi-T_1 space in a topological space was first introduced by the mathematician S. N. Maheshwari & R. Prasad[3] in the year 1975. The term Semi-open sets are introduced by N. Levine[1] in 1963. Then after the mathematician S. N. Maheshwari & R. Prasad[3] used Semi-open sets to define and introduced the Separation axioms called Semi-Separation axioms like Semi-T_0 space in the year 1975. Later, the mathematician P. Bhattacharya & B. K. Lahiri[6] used to Semi-open sets to define the axiom Semi-$T_{1/2}$ space in the year 1982 and further investigated the separation axioms like Semi-T_0 space, Semi-T_1 space, Semi-T_2 space, Semi-T_3 space, Semi-T_4 space, Semi-T_5 space. The mathematician Charles Dorsett[5] introduced the concept of Semi-Regular & Semi-Normal spaces and investigated their properties.

In this paper, analogous to S. N. Maheshwari & R. Prasad's[3] Spaces, we investigate the certain results of these spaces in the topological space.

II. Preliminaries

Throughout this paper (X , τ) is always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X , τ) then $C_L(A)$ & $I_N(A)$ are denote the closure and interior of the set A in the topological space.

2.1 . Semi- T_0 space[3] : A topological space (X , J) is said to be Semi-T_0 space if and only if given any pair of distinct points x & y of set X, ∃ a Semi-nbd of one of them not containing the other.

Or, A topological space (X , J) is Semi-T_0 space if and only if given any pair of distinct points x , y of X , ∃ a Semi-open set of one of them not containing the other.

Or, A topological space (X , J) is Semi-T_0 space if and only if ∃ a Semi-open set G such that either $x \in G$ & $y \notin G$ or ∃ a Semi-open set H such that $x \notin H$ & $y \in H$.

2.2 . Proposition : Every Discrete space is a Semi-T_0 space in the topological space.

Verification : Let (X , D) be a Discrete space and let x & y be the distinct points of X. Since, every subset of Discrete space is an open set and every open set is a Semi-open set in topological space. So, {x} is a Semi-open set $G \in J$ such that $x \in G$ but $y \notin G$ in J.

Hence, every Discrete space is a Semi-T_0 space in the topological space.

2.3 . Proposition : Every Indiscrete space is not a Semi-T_0 space in the topological space.

Verification : Let (X , I) be a Indiscrete space and let x & y be two distinct points of set X. Since, every subset of Indiscrete space is an open set and every open set is a Semi-open set in topological space but the only Semi-open set of the point ' x' is X and which also contains the point 'y' so ∄ a Semi-open set of the point 'x' which does not contains the point 'y'.

Hence, every Indiscrete space is not a Semi-T_0 space in the topological space.

2.4 . Proposition : Let J and J^* be two topologies on a nonempty set X and let J^* be finer than J, that is $J \subset J^*$ then if J is Semi-T_0 space then J^* is also a Semi-T_0 space.

Proof : Let x and y be any two distinct points of the set X. Since, (X , J) is Semi-T_0 space so, ∃ a Semi-open set $G \in J$ such that $x \in G$ but $y \notin G$ in J. Since, $J \subset J^*$ so G is also a Semi-open set such that $x \in G$ but $y \notin G$ in J^*. **Hence, (X , J^*) is a Semi-T_0 space.**

2.5 . Illustration : Consider the set X = {a , b} and the topology J = { φ ,{a}, X } then the topological space (X , J) is a Semi-T_0 space.

Since, (X , J) is a Semi-T_0 space as for given a & $b \in X$. So, ∃ $\{a\} \in J$ such that $a \in \{a\}$ & $b \notin \{a\}$. **Hence, (X , J) is a Semi-T_0 space.**

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2.6 . Pre-Semi open function : Let (X , J_1) and (Y , J_2) be two topological spaces and consider the function $f : (X , J_1) \longrightarrow (Y , J_2)$ then the function f is said to be Pre-Semi open function if f maps each Semi-open set of (X , J_1) to a Semi-open set of (Y , J_2).

2.7 . Semi-homeomorphism : Let (X , J_1) and (Y , J_2) be topological spaces and consider the function $f : (X , J_1) \longrightarrow (Y , J_2)$ then the function f is said to be a Semi-homeomorphism of (X , J_1) onto (Y , J_2) if the function f is one-one and onto, irresolute & Pre-Semi open function.

2.8. Semi-homeomorphic : A topological spaces (X , J_1) is Semi-homeomorphic with other topological spaces (Y , J_2) if $\exists$ a Semi-homeomorphism of (X , J_1) onto (Y , J_2).

Since, each homeomorphism of (X , J_1) onto (Y, J_2) is a Semi-homeomorphism of (X , J_1) onto (Y, J_2) but not conversely.

2.9 . Semi-topological property : A property of a topological space X is said to be Semi-topological property if and only if it is preserved under Semi-homeomorphism.

2.10. Proposition : The property of a topological space being a Semi-T_0 space is preserved under one-one & onto Pre-Semi open function and hence is a Semi-topological property.

Proof : Let (X , J) be a Semi-T_0 space and let f be a one-one Pre-Semi open function of topological space (X , J) onto another topological space (Y , J^*) then we have to show that (Y , J^*) is also a Semi-T_0 space.

Let y_1 & y_2 be any two distinct points of the set Y. Since, the mapping f is one-one & onto. So, $\exists$ distinct points x_1 & x_2 of the set X such that $f(x_1) = y_1$ & $f(x_2) = y_2$.

Since, (X , J) be a Semi-T_0 space. So, $\exists$ a Semi-open set 'G' $\in J$ such that $x_1 \in G$ but $x_2 \notin G$.

Since, mapping f is a Pre-Semi open function. So, $f(G) \in J^*$ is a Semi-open set such that $y_1 = f(x_1) \in f(G)$ but $y_2 = f(x_2) \notin f(G)$.

Thus, $f(G)$ is a Semi-open set of y_1 & not containing y_2.

Hence, (Y , J^*) is also a Semi-T_0 space.

Since, the property of being Semi-T_0 space is preserved under one-one & onto Pre-Semi open function. So, it is preserved under Semi-homeomorphism.

Hence, it is a Semi-topological property.

Remark : - Moreover, according to [3] the implications hold :

$$\text{“ } \mathbf{T_2\text{-space}} \longrightarrow \mathbf{Semi\text{-}T_2\text{-space}}$$
$$\downarrow \qquad\qquad \downarrow$$
$$\mathbf{T_1\text{-space}} \longrightarrow \mathbf{Semi\text{-}T_1\text{-space}}$$
$$\downarrow \qquad\qquad \downarrow$$
$$\mathbf{T_0\text{-space}} \longrightarrow \mathbf{Semi\text{-}T_0\text{-space}} \text{ ”}$$

2.11 . Theorem : A topological space (X , J) is a Semi-T_0 space if and only if for any distinct arbitrary points x & y of the set X, the Semi-closure of {x} & {y} are distinct.

Proof : Let the two distinct points x & y of the set X.

Since, $x \neq y \Rightarrow SC_L\{x\} \neq SC_L\{y\}$, where SC_L is the Semi-closure.

Since, $SC_L\{x\} \neq SC_L\{y\}$ so ∃ at least one point 'z' of the set X which belongs to one of them, say $SC_L\{x\}$ and does not belong to $SC_L\{y\}$. ------- (I)

We claim that, $x \notin SC_L\{y\}$.

Let $x \in SC_L\{y\}$ then $SC_L\{x\} \subset SC_L\{y\}$. So, $x \in SC_L\{x\} \subset SC_L\{y\}$ which contradict (I).

Thus, $x \notin SC_L\{y\}$ and consequently $x \in (X - SC_L\{y\})$.

Also since, $SC_L\{y\}$ is Semi-closed set so $(X - SC_L\{y\})$ is Semi-open set.

Hence, $(X - SC_L\{y\})$ is a Semi-open set of the point x and not containing the point 'y'.

Thus, (X , J) is a Semi-T_0 space.

Conversely,

Let (X , J) be a Semi-T_0 space and let x & y be the distinct points of X then we have to show that $SC_L\{x\} \neq SC_L\{y\}$.

Since, the space is Semi-T_0 space. So, ∃ a Semi-open set G containing one of them say 'x' but not containing the point 'y'. So, (X – G) is a Semi-closed set which does not contain the point 'x' but contain the point 'y' then by definition of Semi-Closure of {y} is the intersection of all Semi-closed set containing {y}. It follows that, $SC_L\{y\} \subset (X - G)$.

Hence, $x \notin (X - G) \Rightarrow x \notin SC_L\{y\}$. Thus, $x \in SC_L\{x\}$ but $x \notin SC_L\{y\}$.

It follows that $SC_L\{x\} \neq SC_L\{y\}$.

Hence, it is proved that the distinct points have distinct Semi-closure in a Semi-T_0 space.

Remark : Every subspace of a Semi-T_0 space is a Semi-T_0 space and so the property is hereditary.

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REFERENCES

[1] Levine, N. (1963). **Semi-open sets and semi-continuity in topological spaces**, *Amer. Math. Monthly*, **70**(1), 39-41.

[2] Caldas M., (1994). **Semi-$T_{1/2}$ Spaces**, Pro Math. **8**, 115-121.

[3] Maheshwari S. N. & Prasad R. (1975). **Some new Separation Axioms,** *Ann. Soc. Sci. Bruxelles*. **89**, 395-402.

[4] Tong J., (1982). **A Separation Axiom Between T_0 and T_1**, *Ann. Soc. Sci. Bruxelles*. **96**, 85-90.

[5] Dorsett C.,(1982). **Product spaces and semi-separation axioms**, *Periodica Math. Hungar.* **13**, 39–45.

[6] Bhattacharyya, P. & Lahiri, B. K. (1987). **Semi-generalized closed set in topology**, *Indian Journal of Math.*, **29**, 375-382.

[7] Dorsett C.,(1982). **Semi-regular spaces**. *Soochow J. Math*. 845–53.

[8] Hamllet N., (1975). **A correction to the paper Semi-open sets and semi-continuity in topological spaces by Norman Levine**, *Proc. Amer. Math. Soc.*, **49**, 458–460.

[9] Jankovic D. S. & Reilly I. L., (1985). **On semi separation properties,** *Indian J. Pure Appl. Math. Sci.* , **16** , 957–964. ´

[10] Munkres J. R.,(1975). **Topology**, *A First Course, Prentice-Hall, Inc.* .

[11] Noiri T., (1976). **On semi-T_2-spaces** , *Ann. Soc. Sci. Bruxelles Ser. I* , **90**, 215–220.

[12] Crosseley S. G. (1978). **A note on semi topological properties**, *Proc. Amer. Math. Soc.* **72**, 409 – 412.

[13] Munshi B. M., (1986). **Separation Axioms**, *Acta Ciencia India*, **12**, 140 -144.

[14] Levine N., (1961). **A decomposition of continuity in topological spaces**, *Amer. Math. Monthly*, **68**, 44-46.

[15] David R. Wilkins & Hilary, (2001). **Topological spaces**, course 212.

[16] K. Priti., **Some Properties of Semi-topological concepts**, TMBU, Bhagalpur, Bihar, India, 812007

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